Algebra and Functions

14 questions	12 questions	17 questions	17 questions	8 questions	12 questions
Number Sense	**Statistics, Data Analysis, Probability**	**Algebra and Functions**	**Measurement and Geometry**	**Math Reasoning**	**Algebra I**

CAHSEE contains 17 Algebra and Functions items. The Algebra and Functions strand is most closely associated with the representation of quantitative relationships, such as functions, equations, graphs, geometric diagrams, and verbal expressions. As students increase their mathematical knowledge and skills, they work frequently with algebraic symbols, expressions with variables, and graphical representations. It is essential that students develop an understanding of several different meanings and uses of variables through multiple representations. Everyday experiences with linear functions should aid in the development of the concepts of proportionality and the ability to discriminate between linear and nonlinear functions. Students must also learn to recognize and generate equivalent expressions, solve linear equations, and effectively use formulas.

To demonstrate achievement in this strand, students will be asked to:
- work with patterns and relationships.
- represent, analyze, and generalize a variety of patterns with tables, graphs, and symbolic rules.
- compare different forms of representations.
- identify functions.
- use algebraic expressions.
- solve linear equations.

The use of mathematical models to represent and understand quantitative relationships is developed by modeling and solving contextualized problems. The analysis of change in various contexts involves tools such as graphs to analyze the nature of changes in quantities in linear relationships.

The ten California academic content standards covered by the CAHSEE Algebra and Functions strand are discussed in the following pages.

Algebra Functions

Strand	**Algebra and Functions (AF)**

Standard | **7AF1.1**

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

2 test questions

Which of the following inequalities represents the statement, "A number, x, decreased by 13 is less than or equal to 39"?

A $13 - x \geq 39$

B $13 - x \leq 39$

C $x - 13 \leq 39$

D $x - 13 < 39$

M03049

Translating verbal descriptions into mathematical expressions is essential in solving real-world problems. CAHSEE questions in this standard require students to translate between verbal descriptions and mathematical equivalents. Students should be able to use variables and appropriate operations to write or identify an expression, an equation, a system of equations or inequality to solve a problem.

Sample Test Question
The correct answer is choice C. Students should recognize that "a number, x, decreased by 13" is represented as $x - 13$ and that "less than or equal to 39" is represented by ≤ 39. Putting both parts of the statement together, $x - 13 \leq 39$.

Analysis of Distractors

Distractor A: represents "13 decreased by a number, x, and also greater than or equal to 39"

Distractor B: used the correct inequality notation but, like Distractor A, represents "13 decreased by a number, x"

        C: correct answer

Distractor D: represents the appropriate expression for "a number, x, decreased by 13" but represents "less than 39," rather than "less than or equal to 39"

Strand	**Algebra and Functions (AF)**

Standard **7AF1.2**
Use the correct order of operations to evaluate algebraic expressions such as $3(2x+5)^2$.

1 test question

If $h=3$ and $k=4$, then $\dfrac{hk+4}{2}-2=$

A   6

B   7

C   8

D   10

M00052

CAHSEE questions for this standard require students to select and use the correct order of arithmetic operations in evaluating expressions (parentheses, exponents, multiplication, division, addition, subtraction). Students may also be required to evaluate expressions that include the distributive property and other basic properties of real numbers.

Sample Test Question
The correct answer is choice A. Students should first substitute 3 and 4 for h and k, then multiply h by k, add 4, divide by 2, and then subtract 2:

$$\frac{(3)(4)+4}{2}-2=\frac{12+4}{2}-2=\frac{16}{2}-2=8-2=6.$$

Analysis of Distractors

A: correct answer

Distractor B: divided by 2 before evaluating the numerator

Distractor C: divided the product of 3 and 4 by 2 before evaluating the numerator

Distractor D: added 4 before multiplying 3 by 4

Algebra
Functions

Strand **Algebra and Functions (AF)**

Standard **7AF1.5**
Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

3 test questions
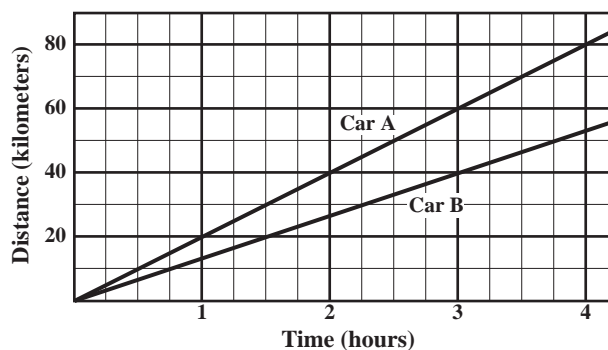
After three hours of travel, Car A is about how many kilometers ahead of Car B?

A 2

B 10

C 20

D 25

M00066

CAHSEE questions for this standard focus on either of its two main components. The first component requires the selection and execution of a graph that accurately and appropriately represents a quantitative relationship. The second component requires the interpretation of information presented in graphical form. All graphs in items from this standard must represent an algebraic relationship.

Sample Test Question
The correct answer is choice C. Students should recognize that distance, in kilometers, is recorded on the *y*-axis, and time, in hours, is recorded on the *x*-axis. Three hours on the time scale corresponds to 60 kilometers for Car A, and three hours corresponds to 40 kilometers for Car B. The number of kilometers that Car A is ahead of Car B after 3 hours is represented by the difference between the distance traveled by Car A and the distance traveled by Car B in the same time $(60 \text{ kilometers} - 40 \text{ kilometers} = 20 \text{ kilometers})$.

Analysis of Distractors

Distractor A: the number of hours that Car A traveled when it had gone 40 kilometers; the distance Car B traveled in 3 hours

Distractor B: made an error when reading the scale of the graph, assumed that the increments had a value of 10

        C: correct answer

Distractor D: used the approximate difference in distance at 4 hours, rather than 3

Strand	**Algebra and Functions (AF)**

Standard   **7AF2.1**

Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

1 test question

$x^3 y^3 =$

A $9xy$

B $(xy)^6$

C $3xy$

D $xxxyyy$

M02879

This standard has four main components: the concept of positive whole-number powers as repeated multiplication, the concept of negative whole-number powers as repeated division, multiplication by the multiplicative inverse, and simplification and evaluation of expressions that include exponents. The first two components of this standard are assessed in the related Number Sense standards 2.1 and 2.3. CAHSEE questions that assess student achievement in this standard may require students to evaluate monomial expressions. Other questions for this standard may require students to demonstrate an understanding of the multiplicative inverse.

Sample Test Question

The correct answer is choice D. Students should recognize that $x^3 = x \bullet x \bullet x$ and that

$y^3 = y \bullet y \bullet y$, so that $x^3 y^3 = x \bullet x \bullet x \bullet y \bullet y \bullet y = xxxyyy$.

Analysis of Distractors

Distractor A: multiplied the exponents and used the product as a coefficient

Distractor B: added the exponents

Distractor C: moved the value of the exponents to serve as a coefficient

        D: correct answer

Algebra
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Strand	**Algebra and Functions (AF)**

Standard **7AF2.2**
Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

1 test question

Simplify the expression shown below.

$$\left(6a^4bc\right)\left(7ab^3c\right)$$

A $\quad 13a^4b^3c$

B $\quad 13a^5b^4c^2$

C $\quad 42a^4b^3c$

D $\quad 42a^5b^4c^2$

M02109

CAHSEE questions in this standard require students to multiply and divide monomials, expand powers, and find roots for monomials when the results are integer exponents. Items may also include combinations of multiplying and dividing monomials. Students should be comfortable with the rules for multiplying and dividing exponential expressions with the same base.

Sample Test Question
The correct answer is choice D, as $6 \bullet 7 = 42$ and the product of a^4 and a is a^5; the product of b and b^3 is b^4; and the product of c and c is c^2.

Analysis of Distractors

Distractor A: added 6 and 7 instead of multiplying and failed to use a as a^1, b as b^1, and c as c^1 when adding the values of the exponents

Distractor B: added 6 and 7 instead of multiplying

Distractor C: multiplied the exponents incorrectly, as in Distractor A
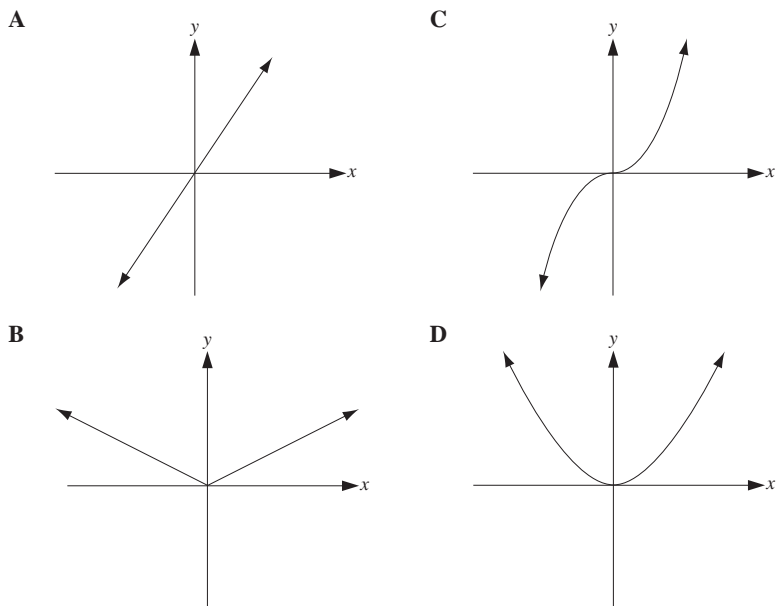
D: correct answer

Strand **Algebra and Functions (AF)**

Standard **7AF3.1**
Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

1 test question

Which of the following could be the graph of $y = x^3$?

A

C

B

D

M02200

CAHSEE questions in this standard require knowledge of graphing basic quadratic functions and cubic functions, as demonstrated by selecting the appropriate graph of a given function or by selecting the appropriate function for a given graph. Some questions may also require knowledge of function graphing to solve problems. As part of their foundational understanding of functions, students should be able to predict the shape of a graph based on the characteristics of the given function (e.g., linear, quadratic).

Sample Test Question
The correct answer is choice C. Students should understand the basic concepts underlying the problem—that cubic functions are nonlinear and that negative values for *x* correspond to negative values for *y* and positive values for *x* correspond to positive values for *y*.

Analysis of Distractors

Distractor A: a linear graph, rather than a nonlinear function

Distractor B: the graph of an absolute value function
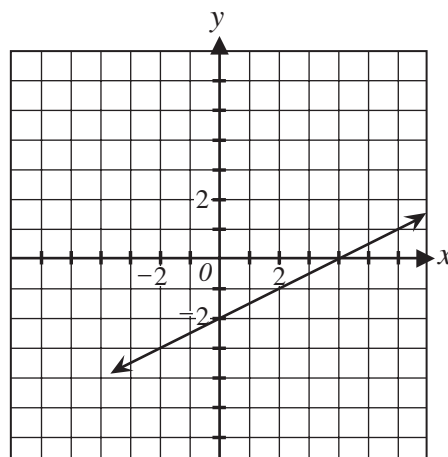
C: correct answer

Distractor D: the graph of a quadratic function

Strand	**Algebra and Functions (AF)**

Standard **7AF3.3**
Graph linear functions, noting that the vertical change (change in *y*-value) per unit of horizontal change (change in the *x*-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

2 test questions

What is the slope of the line shown in the graph above?

A $\quad -2$

B $\quad -\dfrac{1}{2}$

C $\quad \dfrac{1}{2}$

D $\quad 2$

M02556

Students should understand that linear functions can model many real-world phenomena and that the rate of change in a function is shown by the slope of the graph of the function. A conceptual understanding of slope can be a key element in students' development of proportional reasoning skills. CAHSEE questions for this standard may focus on either of its two main components. The first component is graphing linear functions on the *xy*-coordinate system. The second is the identification of the slope in quantitative terms from a given linear function or the selection of a given slope from a numerical value, from a line shown on a graph, or from two pairs of coordinate points.

Sample Test Question
The correct answer is choice C. Students should understand slope as the change in *y* divided by the change in *x* and/or as the ratio "rise over run." In this problem, the change in the *y*-value is obtained by subtracting –2 from 0, and the change in the *x*-value is obtained by subtracting

0 from 4, and thus $\dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{0-(-2)}{4-0} = \dfrac{2}{4} = \dfrac{1}{2}.$

Analysis of Distractors

Distractor A: the value of the *y*-intercept, misunderstood the concept of slope

Distractor B: made an error in the subtraction of $0-(-2)$ or $4-0$

C: correct answer

Distractor D: divided the change in *x* by the change in *y*

Strand   **Algebra Functions (AF)**

Standard   **7AF3.4**
Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of a line equals the quantities.

1 test question

The graph below shows Francine's electric bill for 4 different months. What is the price per kilowatt-hour of Francine's electricity?

Monthly Electric Bill
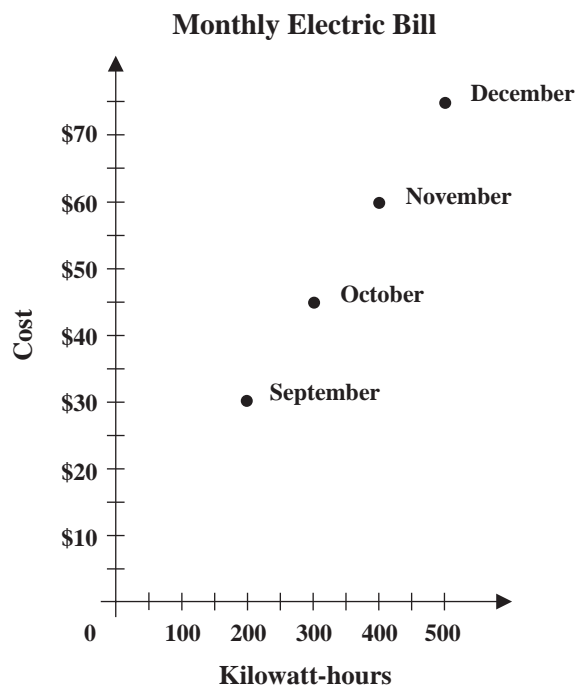
A $0.15

B $0.30

C $1.50

D $6.67

M02681

Graphing direct variation is a powerful way to comprehend and express proportional reasoning. CAHSEE questions within this standard focus on either of its two main components, both of which require students to understand the relationship between the graphical presentation of data and the symbolic representation of data. The first component involves the identification of the correct graph. The second component involves the determination of the slope of a direct variation and the interpretation of the meaning of the slope as a constant ratio between the two quantities in the variation.

Sample Test Question

The correct answer is choice A. Students must understand the relationship between the kilowatt-hours on the *x*-axis and the corresponding cost on the *y*-axis. After studying the given data points, the students should recognize that the relationship is linear. The students should then divide the *y*-value by the *x*-value for any of the given points (September, October, November, or December). The result will be a constant $0.15.

Analysis of Distractors

A: correct answer

Distractor B: used the incorrect *x*-value in the calculation $\left(\$30 \div 100 = \$0.30\right)$

Distractor C: made a decimal error in the calculation

Distractor D: reversed the correct order by dividing the *x*-value by the *y*-value $\left(200 \div \$30 = \$6.67\right)$

Algebra Functions

Strand	**Algebra and Functions (AF)**

Standard **7AF4.1**
Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

3 test questions

In the inequality $2x + \$10,000 \geq \$70,000$, x represents the salary of an employee in a school district. Which phrase most accurately describes the employee's salary?

A   At least $30,000

B   At most $30,000

C   Less than $30,000

D   More than $30,000

M02621

CAHSEE questions for this standard may focus on any of its components, including solving two-step linear equations, solving two-step inequalities, interpreting the solutions of equations or inequalities, and judging the reasonableness of the solutions of equations or inequalities.

Sample Test Question
The correct answer is choice A. Students should recognize that the inequality has the solution $x \geq \$30,000$ and that the correct way to state this inequality is "at least $30,000."

Analysis of Distractors
The distractors offer incorrect solutions for the inequality and/or ways to express the mathematical notation.

Strand	**Algebra and Functions (AF)**

Stephanie is reading a 456-page book. During the past 7 days she has read 168 pages. If she continues reading at the same rate, how many more days will it take her to complete the book?

Standard **7AF4.2**
Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

2 test questions

A 12

B 14

C 19

D 24

M00380

Problem solving is a significant higher-order thinking skill that enables students to apply their mathematical knowledge to real-world situations. CAHSEE questions for this standard may require students either to solve a specific multistep problem or to determine the equation that should be used to solve the problem. Questions may also require students to understand the concept of direct variation and to recognize that direct variation may also be expressed as a linear function. This standard is closely related to Algebra I standard 15.0, which requires students to solve a variety of problems such as rate, work, and percent mixture using algebraic methods.

Sample Test Question
The correct answer choice is A. One method is to subtract 168 from 456 to obtain the number of pages left. Then set up the proportion $\frac{168}{7} = \frac{288}{x}$ and solve for *x*.

Analysis of Distractors

A: correct answer

Distractor B: the number of pages read per day, assuming 168 pages is the amount of pages left, can be found by $\frac{168}{12}$

Distractor C: the total days that are needed to read the book can be found from the proportion $\frac{168}{7} = \frac{456}{x}$

Distractor D: the number of pages read per day can be found by $\frac{456}{19}$